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IS QUANTUM MECHANICS WITH *CP* NONCONSERVATION INCOMPATIBLE WITH EINSTEIN'S LOCALITY CONDITION AT THE STATISTICAL LEVEL?

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Received 23 October 1987; revised manuscript received 30 March 1988; accepted for publication 9 May 1988
Communicated by J.P. Vigier

As a sequel to our earlier work, we present a general analysis, in terms of density operators, of the EPR-type gedanken situation (in the presence of *CP* noninvariance) involving basis states which are mutually non-orthogonal but partially distinguishable. We also comment on the published criticisms of our earlier work.

In an earlier paper [1] we had pointed out a curious gedanken example of the Einstein–Podolsky–Rosen (EPR) paradox using *CP* nonconservation. The example involves a pair of correlated neutral pseudo-scalar mesons (M^0–$\overline{M^0}$) originating from the decay of a $J^{PC}=1^{--}$ vector meson. The exponentially decaying states with definite masses and lifetimes are denoted by $|M_L\rangle$ and $|M_S\rangle$ (which are certain linear combinations of $|M^0\rangle$ and $|\overline{M^0}\rangle$) and they are used to describe the quantum mechanical time-evolution of the system. In the presence of *CP* noninvariance, $|M_L\rangle$ and $|M_S\rangle$ are non-orthogonal. This non-orthogonality of the physically relevant states (unique characteristic of the quantum mechanical treatment of *CP* nonconservation) was explored in ref. [1]. There it was indicated that in an EPR-type situation involving these states which are partially distinguishable (through certain physical attributes), the quantum mechanical treatment, in principle, implies the possibility of a non-local effect manifesting at the statistical level. This result was obtained by considering a certain transition of the pure state into a mixed state composed of non-orthogonal components. The collapse to such a mixed state was first assumed to be "total" and then the error involved (due to overlap between the probability distributions of the invariant masses of the decay products corresponding to the non-orthogonal states) was estimated. It was argued that the error could be, in principle, made small compared to the measure of the non-local effect.

In a subsequent note, Squires and Siegwart [2] have questioned the result of ref. [1]. However, it is important to note that they consider an inherently different scenario where the collapse of the wave function (eq. (9) of their paper) takes place to a mixed state with mutually orthogonal components (eq. (10) of their paper). This amounts to identifying and distinguishing the mutually orthogonal individual decay-product components (in the context of the example dealt with in ref. [1]). The issue of

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partial distinction between the non-orthogonal states of the total decay products (the crucial element in our treatment) is not addressed to by Squires and Siegwart and what they have considered is essentially the standard case where it is already well known that no non-local effect exists at the statistical level. The argument by Finkelstein and Stapp [3] is in effect similar to that of ref. [2]. It was also claimed by the authors of refs. [2] and [3] that the error analysis in ref. [1], referred to earlier, was ambiguous. They suggested a change in the parameter used in ref. [1] as a measure of the error, which swamped the non-local effect. Absence of a rigorous scheme for estimating the error, therefore, makes the issue unclear.

In this paper we show that the result of ref. [1] can be corroborated by a mode of analysis different from that adopted in our earlier paper. Here we avoid the "ambiguity" mentioned above and directly incorporate the notion of what we call the "partial collapse" (or "partial information") type measurement which implies transition such that the coherence of the original pure state is only partially destroyed.

Taking the cue from ref. [1], the two-particle wave function at the time of production ($t=0$) of the pair is given by

$$|\Psi(t=0)\rangle = (|\mathrm{M_S M_L}\rangle - |\mathrm{M_L M_S}\rangle)/N\,, \qquad (1)$$

where N is a normalization factor and the first (second) member of each pair refers to the left (right) hemisphere. Note that we shall follow closely the notation of ref. [1].

Following the discussion given in ref. [1], the subsequent time evolved wave function can be written in the form

$$|\Psi(t)\rangle = C_1|\mathrm{M_L}\phi_\mathrm{S}\rangle + C_2|\mathrm{M_S}\phi_\mathrm{L}\rangle + C_3|\chi\rangle\,, \qquad (2)$$

where C_1, C_2, C_3 are time-dependent constants, and $|\chi\rangle \sim |\mathrm{M_S M_L}\rangle - |\mathrm{M_L M_S}\rangle$ represents the undecayed piece with $\langle\chi|\chi\rangle = 1$. $|\phi_\mathrm{L}\rangle$ ($|\phi_\mathrm{S}\rangle$) corresponds to the decay products on the right from $|\mathrm{M_L}\rangle$ ($|\mathrm{M_S}\rangle$). It may be noted that in (2) we have not considered those components of the wave function which contain decay products on the left as they are irrelevant for our subsequent discussion which is focused on the flux of $|\overline{\mathrm{M}^0}\rangle$ on the left.

It is clear from eq. (2) that the above flux involves a contribution due to the overlap between the decay product states $|\phi_\mathrm{L}\rangle$, $|\phi_\mathrm{S}\rangle$ (common decay products from $\mathrm{M_L}$ and $\mathrm{M_S}$) on the right, which is non-vanishing in the presence of CP violation [1]. The very fact that the statistical property of the particles on one side has some formal dependence pertaining to the interference between the physical states of the particles on the other side is the key feature of this example. Whether this interference can be physically tampered by suitably selecting the particles on the right is the point at issue. It needs to be noted that these decay product states have physical attributes (e.g. invariant masses, lifetimes of their parent particles, etc.) associated with them. In the absence of CP violation, the states $|\phi_\mathrm{L}\rangle$, $|\phi_\mathrm{S}\rangle$ are orthogonal and they can be distinguished unambiguously according to the ideas of the standard quantum measurement theory. However, if in the presence of a small but non-vanishing CP violating interaction, one can partially discriminate between the states $|\phi_\mathrm{L}\rangle$ and $|\phi_\mathrm{S}\rangle$ by exploiting the differences in their physical attributes, there arises a possibility, in principle, to affect the coherence of the wave function given by eq. (2).

Such a scheme envisages non-orthodox measurements partially destroying the coherence of the original pure state and leading to mixtures of non-orthogonal states. It should be emphasised that the concept of such measurements ("partial collapse") is not prima facie inadmissible and can be dealt with, at least in principle, by suitable generalisation of the standard quantum theory of measurement, as has been shown by various authors [4]. In this context it may be noted that recently Ivanović [5] has analysed the viability of possible non-standard schemes to differentiate between non-orthogonal states. There are various examples of realistic measurements [4] such as approximate measurements and/or measurements with imperfect apparatus which cannot be described by the standard quantum measurement theory based on orthogonal projections only. This aspect has recently been discussed by Ghirardi et al. [6] in the light of ref. [1] [#1].

Now to formalise this discussion we present a density matrix treatment of our gedanken example in terms of a specific ansatz for "partial collapse". In this treatment we explicitly take probability conser-

[#1] For footnote see next page

vation into account, circumventing thereby the objection raised by Lindblad [8] regarding our earlier work. Here we are interested in the total number of $\overline{M^0}$ ($\sim |M_L\rangle - |M_S\rangle$) on the left in two cases: (A) For no measurement performed on the right; (B) After "partial collapse" type measurement pertaining to $|\phi_L\rangle$ and $|\phi_S\rangle$ on the right.

Let us first consider the case (A). The density operator ρ^A_{LR} corresponding to $|\Psi(t)\rangle$ is given by

$$\begin{aligned}\rho^A_{LR} = &|C_1|^2|M_L\phi_S\rangle\langle M_L\phi_S| \\ &+ |C_2|^2|M_S\phi_L\rangle\langle M_S\phi_L| + |C_3|^2|\chi\rangle\langle\chi| \\ &+ C_1^*C_2|M_S\phi_L\rangle\langle M_L\phi_S| + C_1^*C_3|\chi\rangle\langle M_L\phi_S| \\ &+ C_2^*C_1|M_L\phi_S\rangle\langle M_S\phi_L| + C_2^*C_3|\chi\rangle\langle M_S\phi_L| \\ &+ C_3^*C_1|M_L\phi_S\rangle\langle\chi| + C_3^*C_2|M_S\phi_L\rangle\langle\chi| . \end{aligned} \quad (3)$$

The reduced density operator ρ^A_L for the undecayed system on the left is obtained by taking the trace of ρ^A_{LR} over a complete set of orthonormal states of the system on the right. Then using $\langle\phi_S|\phi_L\rangle = \langle\phi_L|\phi_S\rangle^* = \alpha(t)$ (say), $\langle\phi_S|M_{L,S}\rangle = \langle\phi_L|M_{L,S}\rangle = 0$ and $\langle\phi_{L,S}|\phi_{L,S}\rangle = 1-\exp(-\gamma_{L,S}t) = F_{L,S}(t)$, we get

$$\begin{aligned}\rho^A_L = &|C_1'|^2|M_L\rangle M_L| + |C_2'|^2|M_S\rangle\langle M_S| \\ &+ C_1^*C_2\alpha|M_S\rangle\langle M_L| + C_2^*C_1\alpha^*|M_L\rangle\langle M_S| \\ &+ |C_3|^2\rho_L(\chi) , \end{aligned} \quad (4)$$

where

$$\rho_L(\chi) = \sum_{R_i} \langle R_i|\chi\rangle\langle\chi|R_i\rangle$$

with the kets $|R_i\rangle$ forming a complete orthonormal basis for the system on the right and $|C'_{1,2}|^2 = |C_{1,2}|^2F_{S,L}(t)$. Note that $\langle\chi|\chi\rangle = 1$ implies

$$\sum_{L_i} \langle L_i|\rho_L(\chi)|L_i\rangle = 1 ,$$

where the kets $|L_i\rangle$ form a complete orthonormal basis for the system on the left.

Turning now to the case (B), we consider "measurements" on the right pertaining to physical attributes of the non-orthogonal states $|\phi_L\rangle$ and $|\phi_S\rangle$ resulting in "partial collapse" to a mixed state composed of $|\psi_1\rangle$, $|\psi_2\rangle$, $|\psi_3\rangle$, $|\psi_4\rangle$ with the respective probabilities p_1, p_2, p_3, p_4 $(=|C_3|^2)$ where $|\psi_1\rangle = C_1|M_L\phi_S\rangle + C_2|M_S\phi_L\rangle$, $|\psi_2\rangle = |M_L\phi_S\rangle$, $|\psi_3\rangle = |M_S\phi_L\rangle$, and $|\psi_4\rangle = |\chi\rangle$. Note that in the limit of no non-orthogonality (i.e. $\alpha = 0$) there is "total collapse" in which case $p_1 = 0$, $p_2 = |C_1|^2$, and $p_3 = |C_2|^2$.

After the "partial collapse" type measurement, the density operator ρ^B_{LR} is given by

$$\begin{aligned}\rho^B_{LR} = &p_1|\psi_1\rangle\langle\psi_1| + p_2|\psi_2\rangle\langle\psi_2| + p_3|\psi_3\rangle\langle\psi_3| \\ &+ |C_3|^2|\chi\rangle\langle\chi| , \end{aligned} \quad (5)$$

whence the reduced density operator ρ^B_L corresponding to the undecayed system on the left is obtained to be

$$\begin{aligned}\rho^B_L = &(p_1|C_1'|^2 + p_2')|M_L\rangle\langle M_L| \\ &+ (p_1|C_2'|^2 + p_3')|M_S\rangle\langle M_S| \\ &+ p_1C_1^*C_2\alpha|M_S\rangle\langle M_L| + p_1C_2^*C_1\alpha^*|M_L\rangle\langle M_S \\ &+ |C_3|^2\rho_2(\chi) , \end{aligned} \quad (6)$$

where $p'_{2,3} = p_{2,3}F_{S,L}(t)$. If one invokes probability conservation in the "partial collapse" measurement then $\mathrm{Tr}(\rho^A_{LR}) = \mathrm{Tr}(\rho^B_{LR})$, whence we get

$$\begin{aligned}p_2' + p_3' = &(1-p_1)(|C_1'|^2 + |C_2'|^2 \\ &+ C_1^*C_2\alpha\beta + C_2^*C_1\alpha^*\beta) , \end{aligned} \quad (7)$$

where we have used $\langle M_L|M_L\rangle = \langle M_S|M_S\rangle = 1$, and $\langle M_L|M_S\rangle = \langle M_S|M_L\rangle = \beta$. Note that α and β are related and they both vanish in the limit of *CP* conservation.

Now using (7), we obtain from (4) and (6)

$$\begin{aligned}\rho^B_L - \rho^A_L = &[(p_1-1)|C_1'|^2 + p_2'] \\ &\times(|M_L\rangle\langle M_L| - |M_S\rangle\langle M_S|) \\ &-2\beta(p_1-1)\,\mathrm{Re}(C_1^*C_2\alpha)|M_S\rangle\langle M_S| \\ &+(p_1-1)(C_1^*C_2\alpha|M_S\rangle\langle M_L| \\ &+ C_2^*C_1\alpha^*|M_L\rangle\langle M_S|) . \end{aligned} \quad (8)$$

It is transparent from (8) that $\rho^B_L \neq \rho^A_L$ which is a sig-

[#1] In this connection, it is interesting to note that the well-known example of production of particle tracks in a cloud chamber is interpretable as non-orthodox measurement wehere a plane wave collapses into gaussian wave packets (see ref. [7]). Another simple example of non-orthodox measurement is provided by the Stern–Gerlach experiment for spin-1/2 atoms in which the magnetic field is assumed to be very weak and the counters are placed so close together that each of the two separated beams has a finite probability of being registered in both the counters.

nature of non-locality at the statistical level, i.e. the statistical properties of the undecayed system on the left would change due to the "partial collapse" type measurement on the right. In the limit of no non-orthogonality (no CP violation), $\alpha=\beta=0$ and $p_1=0$, $p_2=|C_1|^2$, whence $\rho_L^B=\rho_L^A$.

Now, to be more specific, we compute $\Delta\overline{M^0}$, the difference in the number of $\overline{M^0}$ observed on the left for (A) and (B). Using the relevant formulae given in ref. [1] we obtain from eq. (8)

$$\begin{aligned}\Delta\overline{M^0} &\equiv \langle\overline{M^0}|(\rho_L^B-\rho_L^A)|\overline{M^0}\rangle\\ &=(1-\beta^2)(1-p_1)\,\mathrm{Re}(C_1^*C_2\alpha)\\ &=\tfrac{1}{2}(p_1-1)\beta e^{-\gamma t}(\cos\Delta mt-e^{-\gamma t})\,. \qquad (9)\end{aligned}$$

In the second line we have substituted the actual expressions for C_1, C_2 and α from ref. [1]. Notice that for CP invariance, $\beta=0$ and hence $\Delta\overline{M^0}=0$. The non-local effect, therefore, crucially hinges on the non-orthogonality between the states $|M_L\rangle$ and $|M_S\rangle$. This is the crux of the essential result also obtained in ref. [1]. A comment on the probabilities p_i ($i=1, 2, 3, 4$) is in order. These probabilities can be calculated unambiguously in the limit of CP conservation. In the presence of CP violation their precise values are, however, not calculable due to non-orthogonality of the basis states and we treat them as phenomenological parameters. It is, however, interesting to note that the non-locality at the statistical level (eq. (9)) does not depend on these details and is non-vanishing unless $p_1=1$, a value which is ruled out for obvious reasons (see eq. (7)).

The genesis of this intriguing non-locality at the statistical level lies in the possibility of partially distinguishing the non-orthogonal states $|\phi_L\rangle$; $|\phi_S\rangle$ through their physical attributes. Of course, if one chooses to confine one's attention only to orthodox quantum measurements involving, for example, the invariant masses of the individual decay product components of $|\phi_L\rangle$ and $|\phi_S\rangle$ which are mutually orthogonal, then the non-locality at the statistical level will not be manifested as shown in refs. [2,3]; what we envisage here is a generalised measurement in the sense discussed in ref. [4]. It should involve non-standard measurements in contrast to ideal measurements entailing orthogonal projections. A concrete operational scheme for realising such measurements in the context of the example discussed in this paper needs further examination. For instance, it may be probed whether it is possible to exploit the difference in the life-times of the states $|M_L\rangle$ and $|M_S\rangle$ to select out partially the decay products corresponding to, say, the $|\phi_L\rangle$ state, tinkering thereby, the interference between $|\phi_L\rangle$ and $|\phi_S\rangle$. This would correspond to a non-orthodox "measurement" involving selection of decay products with their time of origin restricted within a specific interval.

Recently Hall [9] and Ghirardi et al. [6] have argued on the basis of the operation-effect formalism (using the first representation theorem [10]) that even for non-orthodox measurements, the reduced density operator for one particle remains unaffected if the measurement is restricted to its partner. They then conclude that the type of collapse envisaged in our example necessarily corresponds to a "measurement" which affects both the particles simultaneously. However, applicability of this abstract argument based on the first representation theorem for all types of non-orthodox measurements needs to be carefully examined before drawing any firm conclusion. Namiki has pointed out to one of us (private discussion) that the many-Hilbert-spaces formulation of the quantum measurement theory [11] appears to provide a suitable framework to deal with "partial collapse" type measurements. This scheme may be studied to analyse the example discussed here.

The curious result discussed in this paper and ref. [1] gives rise to the following questions:

(a) Is the peculiarity of this example essentially due to the incompleteness of the conventional quantum mechanical formalism (with its inherent approximations) used to describe the decaying systems in the presence of CP noninvariance?

(b) Does this example indicate that the notion of non-orthodox measurement, by itself, can lead to a new feature of the EPR paradox?

One of us (DH) wishes to thank J. Corbett and S. Sengupta for helpful remarks. DH also gratefully acknowledges valuable discussions with the participants at the International Conference to commemorate Schrödinger's birth centenary held at Delphi, Greece. AD thanks the Alexander-von-Humboldt-Stiftung, Bonn, for financial support. The

research of AR is supported by the Indian National Science Academy and the University Grants Commision.

Note added. The specific model for "partial collapse" (partial loss of coherence of the original pure state involving non-orthogonal components) used in this paper envisages transition from the pure state given by eq. (2) into a mixed state composed of $|\psi_1\rangle$, $|\psi_2\rangle$, $|\psi_3\rangle$, $|\psi_4\rangle$ (eq. (5)), presuming the possibility that at least for some of the events, the states $|\phi_L\rangle$ and $|\phi_S\rangle$ can be distinguished so that for the selected sub-ensembles designated by the states $|\psi_3\rangle$ and $|\psi_2\rangle$ the projection operators corresponding to the states $|\phi_L\rangle$ and $|\phi_S\rangle$ have definite values $(=+1)$. For simplicity, we have taken the coefficients C_1, C_2 in $|\psi_1\rangle$ to be the same as those in eq. (2) but this is not essential for the non-local effect obtained in our treatment.

After submission of the manuscript, our attention has been drawn to the paper by Clifton and Redhead [12] related to ref. [1], which like refs. [2] and [3] emphasizes trhat within the framework of the standard theory of measurements in quantum mechanics there is no scope for non-local effect of the type discussed in our paper. What we contend is that since the quantum mechanical treatment of *CP* non-invariance provides an example of physically relevant non-orthogonal states for which one may consider applying the notion of "partial distinction" (exploiting the differences in their physical attributes), it raises the issue of "non-orthodox" measurements in the context of the EPR example – an arena hitherto left unexplored and which lies beyond the ambit of the standard theory of quantum measurements. "Partial distinction" between non-orthogonal states, in a sense, involves "unsharp" or "imprecise" simultaneous measurement of noncommuting observables, a concept whose tenability has been analysed by various authors like those mentioned in refs. [4,5] and also by Wootters and Zurek [13], Busch [14], Mittelstaedt et al. [15] and Greenberger and Yasin [16]. One may also note here the recent papers by Dieks [17] and Corbett [18] which study in depth the particular question of "partial discrimination" between non-orthogonal states. Possible limitations of the operation-effect formalism (discussed in refs. [6,9,10]) in order to cover all possible models of non-orthodox measurements pertaining to non-orthogonal states have been critically examined by Srinivas and Home [19].

In the light of all these studies it is hoped that the model for "partial collapse" used in this paper can be further refined and made more concrete with reference to specific measuremental procedures. This would help to clarify the issue whether the peculiarity of the example discussed in our paper is an indicative of a genuine non-local effect predicted by quantum mechanics at the statistical level.

References

[1] A. Datta, D. Home and A. Raychaudhuri, Phys. Lett. A 123 (1987) 4.
[2] E. Squires and D. Siegwart, Phys. Lett. A 126 (1987) 73.
[3] J. Finkelstein and H.P. Stapp, Phys. Lett. A 126 (1987) 159.
[4] I. Block and D.A. Burba, Phys. Rev. D 10 (1974) 3206;
K. Kraus in: Foundations of quantum mechanics and ordered linear spaces, eds. A. Hartkamper and H. Neumann (Springer, Berlin, 1974);
E.B. Davies, Quantum theory of open systems (Academic Press, New York, 1976);
H.P. Yuen, Phys. Lett. A 91 (1982) 101; in: Proc. 2nd Int. Symp. on Foundations of quantum mechanics (Phys. Soc. Japan, Tokyo, 1987).
[5] I.D. Ivanović, Phys. Lett. A 123 (1987) 257.
[6] G.C. Ghirardi, R. Grassi, A. Rimini and T. Weber, preprint, University of Trieste, Italy, 1987.
[7] W. Heisenberg, The physical principles of the quantum theory (University of Chicago Press, Chicago 1930);
L.I. Schiff, Quantum mechanics (McGraw-Hill, New York, 1968) p. 335.
[8] G. Lindblad, Phys. Lett. A 126 (1987) 71.
[9] M.J.W. Hall, Phys. Lett. A 125 (1987) 89.
[10] K. Kraus, States, effects and operations (Springer, Berlin, 1983).
[11] S. Machida and M. Namiki, in: Proc. 1st Int. Symp. on Foundations of quantum mechanics, eds. S. Kamefuchi et al. (Phys. Soc. of Japan, Tokyo, 1984);
M. Namiki, Ann. N.Y. Acad. Sci. 480 (1986) 78.
[12] R.K. Clifton and M.L.G. Redhead, Phys. Lett. A 126 (1988) 295.
[13] W.K. Woottters and W.H. Zurek, Phys. Rev. D 19 (1979) 473.
[14] P. Busch, Phys. Rev. D 33 (1986) 2253.
[15] P. Mittelstaedt et al., Found. Phys. 17 (1987) 391.
[16] D.M. Greenberger and A. Yasin, Phys. Lett. A.128 (1988) 391.
[17] D. Dieks, Phys. Lett. A 126 (1988) 303.
[18] J. Corbett, Phys. Lett. A 130 (1988), to be published.
[19] M.D. Srinivas and D. Home, University of Madras preprint.